Pearson Edexcel

Mark Scheme (Results)

January 2019

Pearson Edexcel International Advanced Level In Mechanics M2 (WME02/01)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL IAL MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes.

   - bod – benefit of doubt
   - ft – follow through
   - the symbol $\surd$ will be used for correct ft
   - cao – correct answer only
   - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC: special case
   - oe – or equivalent (and appropriate)
   - d… or dep – dependent
   - indep – independent
   - dp decimal places
   - sf significant figures
   - $*$ The answer is printed on the paper or ag- answer given
   - $\sqsubset$ or d… The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
    - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
    - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Mechanics Marking

(But note that specific mark schemes may sometimes override these general principles)

- Rules for M marks: correct no. of terms; dimensionally correct; all terms that need resolving (i.e. multiplied by cos or sin) are resolved.

- Omission or extra g in a resolution is an accuracy error not method error.

- Omission of mass from a resolution is a method error.

- Omission of a length from a moments equation is a method error.

- Omission of units or incorrect units is not (usually) counted as an accuracy error.

- DM indicates a dependent method mark i.e. one that can only be awarded if a previous specified method mark has been awarded.

- Any numerical answer which comes from use of g = 9.8 should be given to 2 or 3 SF.

- Use of g = 9.81 should be penalised once per (complete) question.

  N.B. Over-accuracy or under-accuracy of correct answers should only be penalised *once* per complete question. However, premature approximation should be penalised every time it occurs.

- Marks must be entered in the same order as they appear on the mark scheme.

- In all cases, if the candidate clearly labels their working under a particular part of a question i.e. (a) or (b) or (c),……then that working can only score marks for that part of the question.

- Accept column vectors in all cases.

- Misreads – if a misread does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, bearing in mind that after a misread, the subsequent A marks affected are treated as A ft

- Mechanics Abbreviations

  M(A) Taking moments about A.

  N2L Newton's Second Law (Equation of Motion)

  NEL Newton's Experimental Law (Newton's Law of Impact)

  HL Hooke's Law

  SHM Simple harmonic motion

  PCLM Principle of conservation of linear momentum

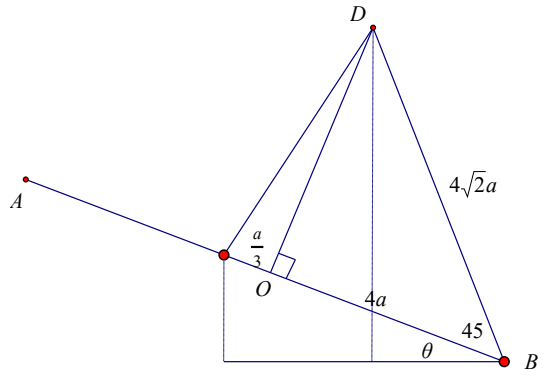
  RHS, LHS Right hand side, left hand side.

January 2019
Mechanics 2 – WME02
Mark Scheme

Q	Scheme	Marks	Notes
1	Moments about x-axis (or parallel axis)	M1	Require all terms. Dimensionally correct. Condone sign errors
	$3m\times 8+m\times 0+2m\times -2=6m\times 2k$	A1	Correct unsimplified equation
	Moments about y-axis (or parallel axis)	M1	Require all terms. Dimensionally correct. Condone sign errors
	$3m\times a+m\times -4+2m\times 5=6m\times k$	A1	Correct unsimplified equation
	Eliminate k and solve for a: $6a+12=20$, $a=\frac{4}{3}$	A1	Or equivalent. 1.3 or better
		(5)	
1alt	Form vector equation in k and a	M1	Require all terms. Dimensionally correct. Condone sign errors
	$3m\begin{pmatrix}a\\8\end{pmatrix}+m\begin{pmatrix}-4\\0\end{pmatrix}+2m\begin{pmatrix}5\\-2\end{pmatrix}=6m\begin{pmatrix}k\\2k\end{pmatrix}$	A1	Or equivalent. Correct unsimplified equation
	Use components from dimensionally correct equation to form two separate equations	M1	Seen or implied Condone processing errors
	$\Rightarrow 3a+6=6k$ $20=12k$	A1	Pair of correct unsimplified equations
	Eliminate k and solve for a: $6a+12=20$, $a=\frac{4}{3}$	A1	1.3 or better
		(5)	
		[5]	

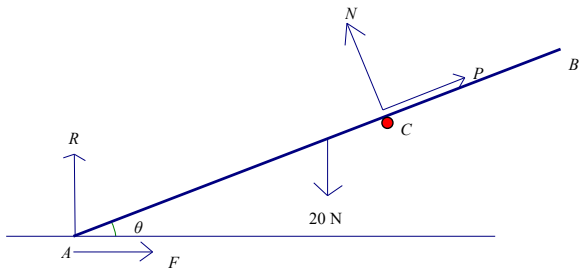
Q	Scheme	Marks	Notes
2a	Use of $\mathbf{I} = m\mathbf{v} - m\mathbf{u}$:	M1	Accept $\pm m(\mathbf{v}-\mathbf{u})$
	$\frac{3}{4}\mathbf{v} = \frac{3}{4}(4\mathbf{i}+\mathbf{j}) + (-6\mathbf{i}+4\mathbf{j})\left(=\left(-3\mathbf{i}+\frac{19}{4}\mathbf{j}\right)\right)$	A1	Correct unsimplified equation
	$\mathbf{v} = \frac{4}{3}\left(-3\mathbf{i}+\frac{19}{4}\mathbf{j}\right) = -4\mathbf{i}+\frac{19}{3}\mathbf{j}$ (m s^{-1})	A1	$-4\mathbf{i}+6.3\mathbf{j}$ (m s^{-1}) or better. ISW Accept as a column vector.
		(3)	
2b	Change in direction	M1	Use **u** and their **v** to find a relevant angle between the two velocities (e.g.not just $\tan^{-1}\frac{1}{4}$)
	$=180° - \tan^{-1}\frac{1}{4} - \tan^{-1}\frac{19}{12}$ or $\tan^{-1}4 + \tan^{-1}\frac{12}{19}$	A1ft	Correct unsimplified. Follow their **v** $(180° - 14.036...° - 57.724...°)$
	$=108°$ $(108.2°)$ (1.89 radians)	A1	108.2° or better. Accept 252°
		(3)	
2b alt	Using scalar product:	M1	Using vectors **u** and their **v** or equivalent
	$\cos\theta = \frac{-16+\frac{19}{3}}{\sqrt{17}\sqrt{16+\left(\frac{19}{3}\right)^2}}$	A1ft	follow their **v**
	$\theta = 108°$ $(108.2°)$	A1	(1.89 radians) Accept 252°
		(3)	
2b alt	Use cosine rule in triangle with sides $\sqrt{17}$, $\frac{\sqrt{505}}{3}$, $\frac{8\sqrt{13}}{3}$	M1	Or equivalent
	$\cos\theta = \frac{17+\frac{505}{9}-\frac{64\times13}{9}}{2\sqrt{17}\frac{\sqrt{505}}{3}}\left(=-0.313\right)$	A1	
	$\theta = 108°$ $(108.2°)$	A1	(1.89 radians) Accept 252°
		(3)	
		[6]	

Q	Scheme	Marks	Notes
3	Either equation of motion	M1	All terms required.
	Motion up the road: $F_U = R + 900g\sin\theta$	A1	One equation correct unsimplified
	Motion down the road: $F_D = R - 900g\sin\theta$	A1	Both equations correct unsimplified
	Use of $P = Fv$ to form at least one equation in R and v	M1	
	$\frac{10800}{v} = R + 900g\sin\theta$ $\frac{10800}{2v} = R - 900g\sin\theta$	A1	Correct unsimplified $(90g\sin\theta = 180)$
	$\left.\begin{array}{l}\frac{10800}{v} = R + 900g\sin\theta \\ \frac{10800}{2v} = R - 900g\sin\theta\end{array}\right\} \Rightarrow \quad \frac{10800}{2v} = 1800g \times \frac{1}{49}$	DM1	Solve simultaneous equations, both containing R and v, for R or v Dependent on 2 preceding M marks
	$v = 15$ only	A1	One correct value
	$R = 540$ only	A1	Both values correct
	Alternative working: $2(R - 900g\sin\theta) = R + 900g\sin\theta$		
	$\Rightarrow R = 540$, $v = 15$		
		(8)	
		[8]	

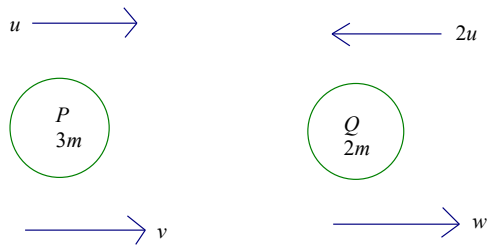
Q	Scheme	Marks	Notes
4a	<table><tr><td></td><td>large</td><td>small</td><td>L</td></tr><tr><td>Mass ratio</td><td>16π</td><td>4π</td><td>12π</td></tr><tr><td>c of m from B</td><td>$4a$</td><td>$3a$</td><td>x</td></tr></table>	B1	Correct ratios and distances
	Moments about B: $16\pi \times 4a - 4\pi \times 3a = 12\pi \times x$	M1	Or equivalent from another point. Require all terms. Condone sign errors. Dimensionally correct.
		A1	Correct unsimplified
	$x = \dfrac{64a - 12a}{12} = \dfrac{52a}{12} = \dfrac{13}{3}a$ ***Given Answer***	A1	Obtain **given answer** from correct working
		(4)	
4b			For all alternative moments equations the distances must relate to the stated point
	Moments about B: $\dfrac{13}{3}a\cos\theta \times M = 4\sqrt{2}a\cos(45+\theta) \times M(1+k)$	M1	Require all terms. Dimensionally correct. Condone sign errors
		A1	Unsimplified equation with at most one error
		A1	Correct unsimplified
	Substitute for trig and solve: $\dfrac{13}{3} \times \dfrac{4}{5} = (1+k) \times 4\sqrt{2}\left(\dfrac{1}{\sqrt{2}} \times \dfrac{4}{5} - \dfrac{1}{\sqrt{2}} \times \dfrac{3}{5}\right)$	DM1	
	$\dfrac{13}{3} = 1+k$, $k = \dfrac{10}{3}$	A1	(3.3 or better)
		[5]	
4b alt	Moments about B:	M1	Require all terms. Dimensionally correct. Condone sign errors
	$\dfrac{13}{3}a\cos\theta \times M = a\cos\theta \times M(1+k)$	A1	Unsimplified equation with at most one error
		A1	Correct unsimplified
	(Substitute for trig and) solve for k: $\dfrac{13}{3} = 1+k$,	DM1	
	$k = \dfrac{10}{3}$	A1	(3.3 or better)

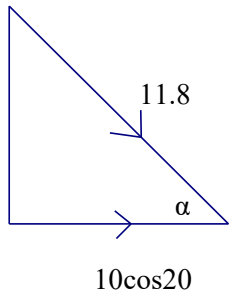
Q	Scheme	Marks	Notes
4b alt	Moments about O:	M1	Require all terms. Dimensionally correct. Condone sign errors
	$-\frac{1}{3}a\times M+kM\times 4a=(kM+M)\bar{x}$	A1	Unsimplified equation, at most one error
	$(k+1)\bar{x}=\left(4k-\frac{1}{3}\right)a$	A1	Correct unsimplified
	$\frac{\bar{x}}{OD}=\frac{3}{4}$	DM1	
	$k=\frac{10}{3}$ (3.3 or better)	A1	
4b alt	New c of m at G where $\frac{OG}{OD}=\tan\theta$, $OG=3a$	M1	
	Moments about G: $Mg\left(3a+\frac{a}{3}\right)=kMg(4a-3a)$	DM1	Require all terms. Dimensionally correct. Condone sign errors
		A1	Unsimplified equation, at most one error
		A1	Correct unsimplified
	$k=\frac{10}{3}$ (3.3 or better)	A1	
4b alt	New c of m at G where $\frac{OG}{OD}=\tan\theta$, $OG=3a$	M1	
	Moments about D: $Mg\left(3a+\frac{a}{3}\right)\cos\theta=kMg(4a-3a)\cos\theta$	DM1	Require all terms. Dimensionally correct. Condone sign errors
		A1	Unsimplified equation, at most one error
		A1	Correct unsimplified
	$k=\frac{10}{3}$ (3.3 or better)	A1	
4b alt	Moments about G: $M\left(\frac{a}{3}+OG\right)=kM(4a-OG)$	M1	Require all terms. Dimensionally correct. Condone sign errors
		A1	Unsimplified equation, at most one error
	$OG(1+k)=a\left(4k-\frac{1}{3}\right)$, $\left(OG=\frac{a(12k-1)}{3(k+1)}\right)$	A1	Correct unsimplified
	$\frac{OG}{OD}=\frac{3}{4}$,	DM1	
	$k=\frac{10}{3}$	A1	
	Other alternatives:		Moments equation: M1A1A1
			Use angle and solve for k: M1A1
		[9]	

Q	Scheme	Marks	Notes
5	Differentiate to find a:	M1	Powers going down
	$a = \frac{\mathrm{d}v}{\mathrm{d}t} = 3t^{\frac{1}{2}} - 6$	A1	
	Solve for $a = 0$:	M1	
	$t^{\frac{1}{2}} = 2 \Rightarrow t = 4$	A1	
	Integrate to find s: $s = \int v\mathrm{d}t$	M1	Powers going up
	$= \frac{4}{5}t^{\frac{5}{2}} - 3t^2 + 2t\,(+C)$	A1	
	Use limits 0 and their 4: $s = \frac{4}{5} \times 32 - 48 + 8 (= -14.4)$	DM1	Limits used correctly Use of 0 can be implied Dependent on the preceding M1
	Distance $= 14.4$ (m) (14 (m))	A1	Or equivalent. Positive answer required
		(8)	
		[8]	

Q	Scheme	Marks	Notes
6a			
	Moments about A:	M1	Dimensionally correct. Condone sin/cos confusion
	$2.5N = 2\cos\theta \times 20$	A1	Correct unsimplified equation
	$N = \dfrac{2\times\frac{4}{5}\times 20}{2.5} = 12.8$ (N)	A1	Accept $\frac{64}{5}$
		(3)	
6b	Resolve $\updownarrow$: $R + N\cos\theta + P\sin\theta = 20$	M1	1st equation. Dimensionally correct. Condone sin/cos confusion and sign errors
	$\left(R = 9.76 - 0.6P\right)$	A1ft	Correct unsimplified equation in N or their N
	Resolve $\leftrightarrow$: $F + P\cos\theta = N\sin\theta$	M1	2nd equation. Dimensionally correct. Condone sin/cos confusion and sign errors
	$\left(F = 7.68 - 0.8P\right)$	A1ft	Correct unsimplified equation in N or their N
	Use $F = \frac{1}{4}R$:	B1	
	Equation in P only: $7.68 - 0.8P = \frac{1}{4}(9.76 - 0.6P)$ $\left(P = 8.06....\right)$	DM1	(or eliminate P) Dependent on the preceding 2 M marks
	Solve for μ : $P = \mu N$	DM1	Dependent on the preceding M mark
	$\mu = 0.630$, (0.63)	A1	0.63 or better
		(8)	
			See over for alternatives

Q	Scheme	Marks	Notes
6b alt	Moments about C: $20\times0.5\cos\theta+F\times2.5\sin\theta=R\times2.5\cos\theta$	M1	1st equation. Dimensionally correct. Condone sin/cos confusion and sign errors
	$(40+7.5F=10R)$	A1	Correct unsimplified equation
	Resolve parallel rod:	M1	2nd equation. Dimensionally correct. Condone sin/cos confusion and sign errors
	$P+F\cos\theta+R\sin\theta=20\sin\theta\,(=12)$	A1	Correct unsimplified equation
	Use $F=\frac{1}{4}R$: $(8.125R=40,\quad R=4.92...)$	B1	
	Solve for P: $P=12-\frac{R}{4}\times\frac{4}{5}-R\times\frac{3}{5}=12-\frac{4}{5}R=8.06...$	DM1	Dependent on the preceding 2 M marks
	Solve for μ : $P=\mu N$,	DM1	Dependent on the preceding M mark
	$\mu=0.630\quad(0.63)$	A1	
		(8)	
6alt	If the use of moments about C is part of the solution to (a)		
	Moments about C and use : $20\times0.5\cos\theta+F\times2.5\sin\theta=R\times2.5\cos\theta$	M1	From (b) Dimensionally correct. Condone sin/cos confusion and sign errors
	Use of $F=\frac{1}{4}R$	B1	From (b)
	$R=\frac{64}{13},\quad(R=4.92...)$	A1	From (b)
	Resolve perpendicular to the rod:	M1	*From (a)
	$N+R\cos\theta=\mu R\sin\theta+W\cos\theta$	A1	From (a) Correct unsimplified equation
	$N=12.8\quad(\mathrm{N})$	A1	From (a)
	Resolve parallel rod:	M1	*Dimensionally correct. Condone sin/cos confusion and sign errors
	$P+F\cos\theta+R\sin\theta=20\sin\theta\,(=12)$	A1	Correct unsimplified equation
	Solve for P: $P=12-\frac{R}{4}\times\frac{4}{5}-R\times\frac{3}{5}=12-\frac{4}{5}R=8.06...$	DM1	
	Solve for μ : $P=\mu N$,	DM1	
	$\mu=0.630\quad(0.63)$	A1	
			* could use an alternative pair of resolutions
		[11]	

Q	Scheme	Marks	Notes
7a			
	CLM (taking the initial direction of P to be +ve)	M1	All terms needed. Dimensionally correct. Condone sign error(s)
	$3m\times u-2m\times 2u=3mv+2mw\left(=-mu\right)$	A1	Correct unsimplified equation
	Negative total $\Rightarrow v<0$ and $w<0$ or $v<0$ and $w>0$ because $w<0$ and $v>0$ is impossible therefore $v<0$ - P has changed direction ***Given answer***	A1	Alt: solve CLM & impact equation for $v\left(=-\frac{u}{5}(1+6e)\right)$ and use the range of possible values for e to justify **given answer**
		(3)	
7b	Impact law:	M1	Used correctly
	$w-v=e\left(u+2u\right)=3eu$	A1	Correct unsimplified equation Signs consistent with CLM equation Allow M1A1 if seen in (a) and used in (b)
	Solve for kw: $v=w-3eu\ \Rightarrow 3\left(w-3eu\right)+2w=-u$ $5w=9eu-u$	DM1	Dependent on the preceding M1
	Correct inequality for their w: $w>0$ $\Rightarrow 9eu-u>0$	DM1	Form inequality in e Dependent on the preceding M1
	$\left(1\geq\right)e>\frac{1}{9}$	A1	Allow if upper limit not stated. A0 if upper limit incorrect. Condone $1>e$
		(5)	
7c	v and w in terms of u: $5w=\frac{9}{2}u-u=\frac{7}{2}u$, $w=\frac{7}{10}u$	M1	Solve for v or w
	$v=w-\frac{3}{2}u=-\frac{8}{10}u$	A1	Both values correct
	Loss in KE	M1	Accept change in KE Must be using two different masses ($3m$ and $2m$)
	$=\frac{1}{2}\times 3m\times\left(u^2-v^2\right)+\frac{1}{2}\times 2m\times\left(4u^2-w^2\right)$	A1ft	Follow their v, w. Correct unsimplified equation
	$=\frac{3}{2}mu^2\left(1-0.64\right)+mu^2\left(4-0.49\right)=4.05mu^2$	A1	$\left(\frac{81}{20}mu^2\text{ or equivalent}\right)$
		(5)	
		[13]	

Q	Scheme	Marks	Notes
8a	Change in KE: $\frac{1}{2}\times 3\times\left(15^2-10^2\right)\ \ (=187.5)$ (J) Gain in GPE: $3g\times 6\sin 20\ \ (=60.3....)$ (J) Work done against friction: $6F$	B1 B1 B1	One term correct unsimplified Two terms correct unsimplified All three terms correct unsimplified
	Work energy:	M1	Dimensionally correct. All terms needed. Condone sign errors and sin/cos confusion
	$187.5=6F+18g\sin 20$	A1	Correct unsimplified equation
	$F=21.2\ \ (21)$	A1	
		(6)	
8b	Energy:	M1	Dimensionally correct. All terms needed. Condone sign errors and sin/cos confusion
	$\frac{1}{2}\times 3\times 100+3\times 9.8\times 6\sin 20=\frac{1}{2}\times 3\times w^2$	A1	Correct unsimplified equation
	$w=11.8\ \ (12)\ \ (\text{m s}^{-1})$	A1	
			
	Direction : $\cos\alpha=\frac{10\cos 20}{11.84...}$	M1	Use trig. to find a relevant angle
	$37.5°\ \ (37°)$ below the horizontal	A1	
		(5)	
8b alt	Find components and use Pythagoras:	M1	Condone sign errors and sin/cos confusion
	$w=\sqrt{\left((10\sin 20)^2+12g\sin 20\right)+(10\cos 20)^2}$	A1	Correct unsimplified equation $\left(v_x=9.396...,\ \ v_y=7.205...\right)$
	$w=11.8\ \ (12)\ \ (\text{m s}^{-1})$	A1	
	Direction : $\tan\alpha=\frac{\sqrt{(10\sin 20)^2+12g\sin 20}}{10\cos 20}$	M1	Use trig. to find a relevant angle
	$37.5°\ (37°)$ below the horizontal	A1	
		(5)	
			See over for (c)

Q	Scheme	Marks	Notes
8c	Use *suvat* to find height above B:	M1	Complete method
	$(10\sin 20)^2 = 2g \times s \qquad (s = 0.5968....)$	A1	Correct unsimplified equation in s
	Total height $= s + 6\sin 20$	DM1	Dependent on the preceding M1
	$= 2.65\,(2.6)$ (m)	A1	
		(4)	
8c alt	Vertical component of w (by trig or Pythagoras)	M1	
	$= 7.205..$	A1	
	Max ht $= \dfrac{v^2}{2g}$	DM1	Dependent on the preceding M1
	$= \dfrac{7.205^2}{19.6} = 2.65$	A1	
		(4)	
8c alt	Conservation of energy:	M1	Using speed at max ht $= 10\cos 20°$ Need all terms. Condone sign error
	$\frac{1}{2} \times 3 \times (10\cos 20°)^2 + 3gh = \frac{1}{2} \times 3w^2$	A1	Correct unsimplified equation
	$h = \dfrac{w^2 - 100\cos^2 20°}{2g}$	DM1	Substitute for w and solve for h Dependent on the preceding M1
	$= 2.65\,(2.6)$ (m)	A1	
		(4)	
		[15]	